ECE 333 Green Electric Energy

Homework 3 - Solution

P492-7.1:

a.

$$P = \eta \cdot \frac{1}{2}\rho A v^3 = \eta \cdot \frac{1}{2}\rho \cdot \frac{\pi d^2}{4} \cdot v^3 = 30\% \cdot \frac{1}{2} \cdot 1.225 \cdot \frac{\pi 20^2}{4} \cdot 10^3 = 57.7\ kW$$

b. use equation 7.17

$$\rho_{2500m,10^o C} = \frac{353.1}{T}\exp(-0.0342\frac{z}{T}) = \frac{353.1}{273.15+10}\exp(-0.0342\frac{2500}{273.15+10}) = 0.922\ kg/m^3$$

c. use equation 7.18

$$P_{2500m,10^o C} = \eta \cdot \frac{1}{2}\rho_{2500m,10^o C} A v^3 = \frac{\rho_{2500m,10^o C}}{\rho}\eta \cdot \frac{1}{2}\rho \cdot \frac{\pi d^2}{4} \cdot v^3 = \frac{\rho_{2500m,10^o C}}{\rho} P = \frac{0.922}{1.225} \cdot 57.7 = 43.43\ kW$$

P492-7.2:

a. Here α=0.2, based on equation 7.18:

$$\frac{v_{120}}{v_{10}} = (\frac{H_{120}}{H_{10}})^{\alpha}$$

$$v_{120} = v_{10} \cdot (\frac{H_{120}}{H_{10}})^{\alpha} = 5 \cdot (\frac{120}{10})^{0.2} = 8.218\ \ m/s$$

$$p_{120} = \frac{1}{2}\rho {v_{120}}^3 = \frac{1}{2}1.225 \cdot 8.218^3 = 339.9\ W/m^2$$

b.

$$v_{40} = v_{10} \cdot (\frac{H_{40}}{H_{10}})^{\alpha} = 5 \cdot (\frac{40}{10})^{0.2} = 6.598\ \ m/s$$

$$p_{40} = \frac{1}{2}\rho {v_{40}}^3 = \frac{1}{2}1.225 \cdot 6.598^3 = 175.9\ W/m^2$$

c.

$$\frac{p_{120}}{p_{40}} = \frac{339.9}{175.9} = 1.93$$

based on equation 7.20

$$\frac{p_{120}}{p_{40}} = \left(\frac{H_{120}}{H_{40}}\right)^{3\alpha} = 1.93$$

d.

$$\rho_{120m,15^oC} = \frac{353.1}{T}\exp(-0.0342\frac{z}{T}) = 1.225\exp(-0.0342\frac{120}{273.15+15}) = 1.208\ kg/m^3$$

$$p_{120m,15^oC} = \frac{1}{2}\rho_{120m,15^oC}v_{120}{}^3 = \frac{1}{2}1.208\cdot 8.218^3 = 335.2\ W/m^2$$

Because

$$\frac{p_{120m,15^oC}}{p_{120m}} = \frac{335.2}{339.9} = 0.986$$

there is no need to consider the density difference in this problem

Problem a. Compare the total wind energy at 0 ^oC, 1 *atm* of pressure, contained in 1-m^2 surface area under the following wind patterns:

$$\rho = \frac{353.1}{T}\exp(-0.0342\frac{z}{T}) = \frac{353.1}{273.15} = 1.293\ kg/m^3$$

(*i*) 100 *hours* of 10 *m/s* winds

$$energy_1 = \frac{1}{2}\rho Av_{10}{}^3\cdot t_{10} = \frac{1}{2}1.293\cdot 1\cdot 10^3\cdot 100 = 64.65\ \ kWh$$

(*ii*) 50 *hours* of 8 *m/s* winds plus 50 *hours* of 12 *m/s* winds

$$energy_2 = \frac{1}{2}\rho Av_8{}^3\cdot t_8 + \frac{1}{2}\rho Av_{12}{}^3\cdot t_{12} = \frac{1}{2}1.293\cdot 1\cdot 8^3\cdot 50 + \frac{1}{2}1.293\cdot 1\cdot 12^3\cdot 50 = 72.41\ \ kWh$$

Although the average wind speeds of two cases above are the same, the total energy produced in the second case is higher than that of the first one. It tells us the average wind

speed is not an appropriate index for us to judge which wind pattern is better because the wind power is proportional to the v_{wind}^{3}**.**